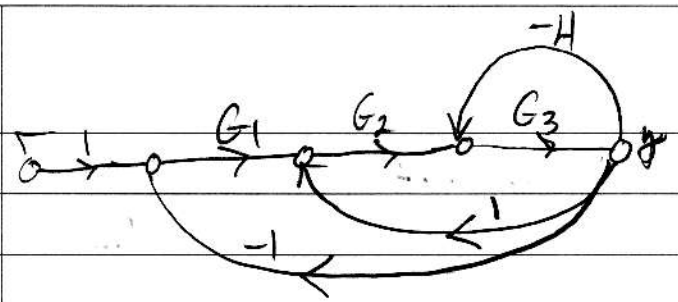
1)



$$\frac{Y}{R} = \frac{\sum P_K \Delta_K}{\Delta}$$

$P_1 = G_1 G_2 G_3$

$L_1 = -G_3 H$

$L_2 = G_2 G_3$

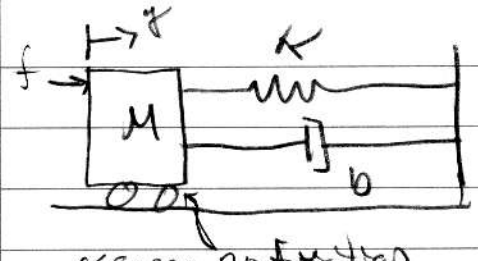
$L_3 = -G_1 G_2 G_3$

$\Delta_1 = 1$

$$\frac{Y}{R} = \frac{G_1 G_2 G_3}{1 - G_2 G_3 + G_3 H + G_1 G_2 G_3}$$

6

2)



assume no friction

Find Y/F

$M = 2.4kg$ $K = 30N/m$ $b = 10Ns/m$.

No initial conditions

$\frac{d^2y}{dt^2} = s^2 Y(s)$ ☆

$\frac{dy}{dt} = sY(s)$

FBD

$$M\frac{d^2y}{dt^2} = F - Ky - b\frac{dy}{dt}$$

$$Ms^2Y(s) = F(s) - KY(s) - bsY(s)$$

$$F(s) = Y(s)[Ms^2 + K + bs]$$

$$\boxed{\frac{Y(s)}{F(s)} = \frac{1}{Ms^2 + bs + K}} = \frac{1}{2.4s^2 + 10s + 30} = \boxed{\frac{0.417}{s^2 + 4.17s + 12.5}}$$

Steadystate

$$\cancel{M\frac{d^2y}{dt^2}} = F - Ky - \cancel{b\frac{dy}{dt}}$$

$$F = Ky$$

$$y = \frac{F}{K} = \frac{1N}{30\frac{N}{m}}$$

$$\boxed{y = 33.3mm}$$

60

$T_s = \frac{4}{\zeta\omega_n}$ since $\zeta = 2\%$

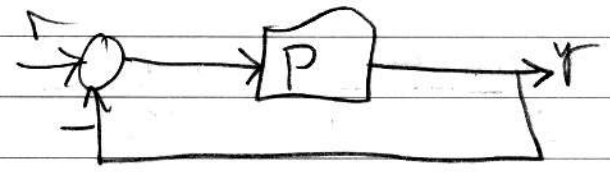
$2\zeta\omega_n = +4.17$ from Tx Fn

$\zeta\omega_n = 2.085$

$T_s = \frac{4}{2.085}$

$$\boxed{T_s = 1.918s}$$

3) Discuss stability & e_{ss} for step & ramp I/P



a) $P(s) = \dfrac{161}{(s+5)(s+4)} = \dfrac{161}{s^2+9s+20}$

$$\frac{y}{r} = \frac{P}{1+P} = \frac{\frac{161}{(s+5)(s+4)}}{1+\frac{161}{(s+5)(s+4)}} = \frac{161\,(s+5)(s+4)}{(s+5)(s+4)\left[(s+5)(s+4)+161\right]}$$

$$\frac{y}{r} = \frac{161}{s^2+9s+181} \qquad s_{1,2} = \frac{-9 \pm \sqrt{(9)^2-4(1)(181)}}{2}$$

$$s_{1,2} = \frac{-9 \pm j\sqrt{643}}{2}$$

System is Stable since $s_{1,2}$ both in LHS plane and no unstable pole zero cancellations.

Step I/p e_{ss}

Type 0 system

$\therefore$ e_{ss} to step is $e_{ss} = \dfrac{1}{1+K_p}$

where $K_p = \lim_{s\to 0} G(s) = \lim_{s\to 0} \dfrac{161}{s^2+9s+20} = \dfrac{161}{20} = 8.05$

$e_{ss} = \dfrac{1}{1+8.05} = 0.11 \Rightarrow \boxed{e_{ss_{\text{step I/p}}} = 11\%}$

Not to bad for stability.

Ramp I/p e_{ss}

Type 0 system

$\therefore$ e_{ss} to ramp I/p is ∞

$\boxed{e_{ss} = \infty}$ System will never reach a steady state with a ramp I/p and stable in that case

$Y(s) = \dfrac{1}{s^2}\dfrac{161}{(s^2+9s+181)}$ using F.V.T. $\lim_{t\to\infty} y(t) = \lim_{s\to 0} s\,\dfrac{161}{s^2(s^2+9s+181)} = \boxed{\infty}$

3)

b) $P(s) = \frac{192}{s(s+5)(s+4)} = \frac{192}{s(s^2+9s+20)} = \frac{192}{s^3+9s^2+20s}$

$T(s) = \frac{Y(s)}{R(s)} = \frac{P(s)}{1+P(s)} = \frac{\frac{192}{s(s+5)(s+4)}}{1+\frac{192}{s(s+5)(s+4)}} = \frac{192}{s(s+5)(s+4)} \cdot \frac{s(s+5)(s+4)}{s(s+5)(s+4)+192}$

$\frac{Y(s)}{R(s)} = \frac{192}{s(s+5)(s+4)+192} = \frac{192}{s^3+9s^2+20s+192}$

Routh Hurwitz

s_3	1	20	0
s_2	9	192	0
s_1	-1.333	0	
s_0	192	0	

$\frac{1(192) - 9(20)}{-1} = -1.333$

System is not stable

Since all left hand column entries don't have the same sign the system has poles in RHS and is unstable.

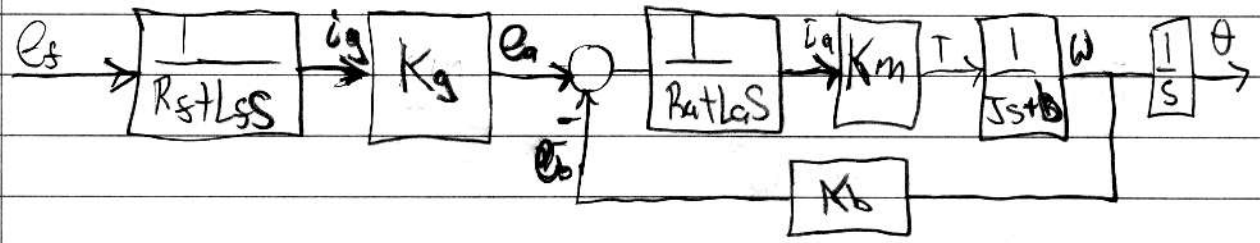
Since unstable you don't care about e_{ss} since it is unstable

6.0

4) Armature controlled DC motor.

generator constant speed ω_g. obtain $\frac{\theta(s)}{e_f(s)}$.

Assume $e_a = K_g i_g$



$$\frac{\theta(s)}{e_f(s)} = \frac{\left(\frac{1}{R_f+L_fs}\right) K_g \left(\frac{1}{R_a+L_as}\right) K_m \left(\frac{1}{Js+b}\right) \frac{1}{s}}{1 + \left(\frac{1}{R_a+L_as}\right) K_m \left(\frac{1}{Js+b}\right) K_b}$$

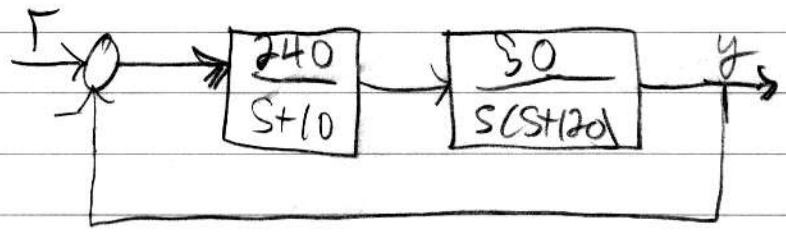
$$\frac{\theta(s)}{e_f(s)} = \frac{K_g K_m}{s(R_f+L_fs)\cancel{(R_a+L_as)}\cancel{(Js+b)}} \; \frac{\cancel{(R_a+L_as)}\cancel{(Js+b)}}{(R_a+L_as)(Js+b)+K_mK_b}$$

$$\boxed{\frac{\theta(s)}{e_f(s)} = \frac{K_g K_m}{s(R_f+L_fs)\left[(R_a+L_as)(Js+b)+K_mK_b\right]}}$$

6.0

(-4.552461535 06, -8.861981661 48)(

5)



Find second order approx.

$$\frac{y}{r} = \frac{\frac{12000}{S(S+10)(S+120)}}{1+\frac{12000}{S(S+10)(S+120}} = \frac{12000}{\cancel{S(S+10)(S+120)}} \cdot \frac{\cancel{S(S+10)(S+120)}}{S(S+10)(S+120)+12000}$$

$$\frac{Y(S)}{R(S)} = \frac{12000}{S^3+130S^2+1200S+12000}$$

Roots

$S_{1,2} = -4.55 \pm j\,8.86 \qquad S_3 = -120.9$

All roots are in LHS => system stable ★

S_3 of -120.9 is $\gg 10(S_{1,2})$ So ignore S portion

we then get:

$$\frac{Y(S)}{R(S)} = \frac{12000}{(S+(4.55+j8.86))(S+(4.55-j8.86))(120.9)}$$

$$\frac{Y(S)}{R(S)} = \frac{99.26}{(S+(4.55+j8.86))(S+(4.55-j8.86))}$$

$S = -\zeta\omega_n \pm j\,\omega_n\sqrt{1-\zeta^2}$

- Find P.O., T_s ($\delta=2\%$), T_p, and e_{ss} to step & ramp I/P's

$$T_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = \frac{\pi}{8.861981661 48} = \boxed{0.355s}$$

$$T_s = \frac{4}{\zeta\omega_n} = \frac{4}{4.552461535 06} = \boxed{0.879s}$$

$\zeta\omega_n = 4.552461535 06$

$\omega_n = \frac{4.552461535 06}{\zeta}$ Next page

Since $\omega_n = \frac{4.55246}{\zeta}$

$\omega_n\sqrt{1-\zeta^2} = 8.86198$

$\frac{4.55246}{\zeta}\sqrt{1-\zeta^2} = 8.86198$

$\left(\zeta\frac{8.86198}{4.55246}\right)^2 = 1-\zeta^2$

$\zeta^2\left(1+\left(\frac{8.86198}{4.55246}\right)^2\right) = 1$

$\zeta = \sqrt{\frac{1}{1+\left(\frac{8.86198}{4.55246}\right)^2}} = \underline{0.4569}$

6.0

$P.O. = e^{\left(\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}\right)} = e^{\left(\frac{-0.4569\pi}{\sqrt{1-(0.4569)^2}}\right)} \times 100$

$\boxed{P.O. = 19.9\%}$

Order of system is 1 so

Step I/P

$e_{ss} = 0$ ✓ $\quad K_p = \lim_{s\to 0} G(s) = \lim_{s\to 0}\frac{240(50)}{(s+10)(s+120)(s)} = \infty$

$e_{ss} = \frac{1}{1+\infty} = 0$

Ramp I/P

$e_{ss} = 1/K_v \quad K_v = \lim_{s\to 0} \frac{s(240)(50)}{s(s+10)(s+120)} = \frac{240(50)}{(120)(10)}$

$e_{ss} = 1/10 = 0.1 \quad = 10$

$\boxed{e_{ss} = 10\%}$ ✓ fairly good steady state error